TECHNICAL NOTES

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COMPARING THE PERFORMANCE OF GEOMETRICALLY SIMILAR AIRPLANES.

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By Max M. Munk.

This note has been prepared for the National Advisory Committee for Aeronautics. It deals with the model rules relating to aeronautical problems, and shows how the characteristics of one airplane can be determined from those of another airplane of different weight or size, and of similar type.

If certain rules for the ratios of the dimensions, the weights and the horsepower are followed, a small low-powered airplane can be used for obtaining information as to performance, stability, controllability and maneuverability of a larger prototype, and contrariwise.

I. It has become common practice to use small airplane models in wind tunnels or in equivalent test arrangements in order to determine the air forces and pressures on an actual airplane in steady flight. The conversion of the model forces to the forces on full size airplanes is performed in accordance with special model rules. There are different kinds of such model rules. The model rule best known is the so-called "square law" rule. The model scale and the airspeed may be chosen arbitrarily, and the actual forces are computed by supposing them to be proportional to the square of the (linear) scale, to the square of the velocity, and to the density of the fluid. This rule is correct only if the viscosity and compressibility of the air, the limited dimensions of the tunnel and some other minor factors do not influence the result. Otherwise, the model rule has to be modified, in order to take into account one or the other of these factors. Several of such special model rules have become known. As a matter of completeness and also as an introduction to the proper subject of this note they will be briefly repeated. This note deals chiefly with the deductions to be drawn from the experience gained with an airplane in actual flight for the prediction of the properties of a second airplane of different size or weight, but geometrically similar. This similarity refers in the first place to the geometrical dimensions of the two airplanes and should also include the shape and particularly the diameter of the propeller. For the study of most questions the two centers of gravity have also to be situated at corresponding points. Some questions require that the length of the radii of inertia and other dimensions, characteristic for the distribution of masses, be included in the similarity.

Such discussion is thought to be particularly desirable at present, since designers of high-powered airplanes give attention to experiences with gliders and low-powered airplanes, and on the other hand, designers of low-powered airplanes make use of experiences gained with high-powered airplanes.

The most convenient way to arrive at any model rule is to consider the physical dimensions of the quantities in question. We know, for instance, that the factors which chiefly govern any forces of a given airplane flying at a given angle of attack or other quantities derived therefrom, are the density of the air ρ, the size of the airplane, characterized by the magnitude b of any characteristic length, (for instance, the span) and the total weight W of the airplane. It will be more convenient to choose as the third factor the load per unit area of the wings, denoted by W/S. The length b, of course, has the dimension of a length and can be measured in feet. The density ρ can be measured in $\text{lb.-sec.}^2\text{-ft.}^{-4}$. The unit load W/S can be measured in lb.ft.^{-2}. Other factors which to a smaller degree influence the air forces will be taken up in detail.

The question arises whether any of such factors, denoted by a, b, c, can be dismissed or disregarded without loss of generality in the information to be gained. Otherwise expressed, the question arises, whether any desired quantity X has to be measured for all possible combinations of the factors a, b, c, governing its size, or whether there are certain rules existing which permit a reduction in the number of factors to be combined in a complete series of tests. This reduction may even be of such an extent that only one test is required, in order that the full information be obtained from this one test by computation.

X can therefore be obtained from a reduced number of tests for all possible combinations of the factors. The great practical importance of this question is at once evident.

Its answer depends upon whether or not there is an essential difference between two different sets of factors, referring, for instance, to model and full-size airplane. The difference noted is not essential if it can be compensated for by the choice of different scales, or if it can only be found by a direct comparison of corresponding factors. A new test is only required if another law of nature becomes manifest, and how can this depend upon the scales used for the measurement? Imagine that no scales of any kind are available, and that two models are situated far from each other, so that no direct comparison of two corresponding factors can be made. The different conditions and factors governing the result are known in each case. It can at once be concluded that the results are equal if the comparison of the different factors of each case with one another (but not the comparison of two factors belonging to the two different cases) can establish no difference in the conditions. Now, such differences, if any, could be expressed in pure ratios only, in numbers giving the ratio of two things of equal kind, these ratios to be derived from the factors only and not by means of measuring the factors by comparing them with scales not essential to the thing itself and arbitrarily brought in.

The answer to the question depends thus on the number of independent combinations of the factors a, b, c, existing, of the form $a^{\alpha}\ b^{\beta}\ c^{\gamma}$, which are pure numbers, α, β, γ, etc. being any exponents. If such combinations exist, the two sets of factors can be distinguished from each other, even without direct comparison, by the value of such numbers. The model rule is then, that such numbers or ratios, if any, will have to be equal in the two cases to be compared, model with airplane for instance, or with two airplanes. If all absolute ratios, derived only from the conditions of the test, agree, the results will also agree, but not absolutely, indeed, only when expressed in units derived in an equal way from the corresponding factors governing the magnitude of the quantity desired. Hence any quantity derived from the test is equal to a certain "coefficient" multiplied by any expression of the form $a^{\alpha}\ b^{\beta}\ c^{\gamma}$, which does not necessarily contain all factors (some exponents might become zero) and whose physical dimension is equal to the physical dimension of the quantity. If the factors a, b, c, are in both cases chosen in a corresponding way, the coefficients will agree for equal numbers derived from the factors, or, otherwise expressed, they will be mathematical functions of these numbers only.

The discussion so far has been rather abstract, but it will become much more tangible when discussed for specific cases in the next paragraphs.

II. I proceed to the discussion of several model rules referring to aeronautical problems. In the simplest and most common case there are only three factors which determine all quantities: they are the density of air ρ, the size of the airplane, expressed by a characteristic length b,* and the unit load W/S. The shape of the airplane, including the propeller dimensions, the angle of attack, and the control settings, are supposed to be invariable. The dimensions of the three factors are:

Length b	unit load W/S	density ρ
Ft.	$lb.ft.^{-2}$	$lb.sec.^2 ft.^{-4}$

The first contains feet only, the second feet and pounds, and the third pounds, feet and seconds. That makes these three factors particularly convenient for the following application.

We first inquire whether there exists any combination $b^{\alpha}(W/S)^{\beta}\rho^{\gamma}$ with at least one finite exponent, which has the dimension zero. There does not. This is at once obvious to a trained mathematician and the reader will try in vain to form such combination. Hence, after what has been said in the first chapter, one single test for a particular angle of attack and setting of all controls, is sufficient to give the magnitude of any quantity in question for any combination of values of the three factors laid down above.

Any air forces, for instance, the lift or drag of the airplane, or a part thereof, has the dimension of a force and can be measured in pounds. In order to obtain the model rule for an air force, we have now to form an expression $b^{\alpha}(W/S)^{\beta}\rho^{\gamma}$ such that its dimension is also that of a force. Since ρ is the only one of the three quantities containing seconds, it cannot occur in such an expression and, therefore, $\gamma = 0$. There remains W/S as the only factor containing pounds. Hence $\beta = 1$. That leaves $\alpha = 2$ and the expression is:

$$\text{Force} = \frac{W}{S}\, b^2 .$$

The method followed shows that this is the only combination of this kind existing.

It follows that any air force is proportional to the product of a coefficient c, which is a pure number and depends upon the shape of the airplane, and $b^2 W/S$.

(1) Air force = $c\, b^2\, W/S$.

In the same way the rules for other quantities can be found. The more important are:

(2) Velocity $V = c\sqrt{\frac{W/S}{\rho}}$

(3) $HP/S = c\,\frac{(W/S)^{3/2}}{\sqrt{\rho}}$

or, substituting (2)

(3') $HP/S = c\;\; W/S\; V$

(4) Time $= c\, b\sqrt{\frac{\rho}{W/S}} = \frac{b}{V}$

(5) Air pressure $= c\;\; W/S = c\, V^2 \rho$.

Substituting (2) into (1) gives

(1') Air force $= c\, b^2\;\; V^2 \rho$.

In the third chapter, the application of these relations will be discussed. It is necessary to finish first the present discussion by proceeding to the cases where additional factors govern the magnitude of the quantities desired. I shall discuss the influence of the viscosity of the air, of its compressibility and the influence of the acceleration of gravity on any airplane maneuver.

The first additional factor to be discussed is the internal friction or viscosity of the air. The ratio of the coefficient of viscosity μ to the density ρ called the coefficient of kinematic viscosity, has the dimension $\frac{\text{area}}{\text{time}}$, ft.^{2}/sec. There can be formed one combination of the three other factors, $b\ V = b\sqrt{\frac{W/S}{\rho}}$ which has the same dimension, and hence, when divided by the coefficient of dynamic viscosity, gives a pure number

$$R = \frac{b\sqrt{W/S}}{\nu\,\rho} = \frac{b\ V}{\nu}$$

called the Reynolds number. It is usual to use this expression and any function of R would serve as well. Hence, if the viscosity has some influence on the things happening, a direct deduction between different airplanes or model and airplane can only be drawn, if in both cases R has the same value. The absolute magnitude of R depends upon the choice of the characteristic velocity and the characteristic length, but if both are chosen in a corresponding way for several cases, the magnitude of R has to agree. The smaller the viscosity, the larger becomes R.

The Reynolds rule requires that in the same medium the velocity be inverse to the scale. The wind tunnel velocity, for instance, should be five times as large as the velocity of flight, if the model is diminished in the scale 1/5. This cannot be done in ordinary atmospheric wind tunnels, as it would require too large velocities and would give too large air pressures and air forces. It is worthy of remark that at equal Reynolds number and in the same medium the magnitude of any air force retains its original value, the force being proportional to the square of both velocity and length, and these being varied inversely to each other so that their product remains constant. The influence of the viscosity is small in many cases, and failure to follow the Reynolds rule has not prevented the wind tunnels from having been the chief source of information regarding air forces on airplanes in steady flight. One wind tunnel only, the variable-density wind tunnel of the National Advisory Committee for Aeronautics, at Langley Field, is now in existence, in which tests with the correct Reynolds number can be made.

Another factor which may have influence on the air forces is the compressibility of the air. It can be characterized by the velocity of sound, about 1100 ft./sec. This suggests at once the ratio of any characteristic velocity velocity to the velocity of sound as the absolute number governing any influence of the compressibility on the properties of airplanes. Incompressible air would possess an infinite velocity of sound, and hence this ratio would be zero. It can be inferred that at a small ratio the influence of the compressibility is small. The ratio of the velocity of flight to the velocity of sound is never large. The only velocity in aeronautics coming near to the velocity of sound is the tip velocity of the propeller blades. Propeller model tests, therefore, should be made with the original tip velocity. This leads to a very high rate of revolution of a small propeller model, but this difficulty can be overcome. An equal tip velocity would give an equal pressure distribution in the same fluid. Hence, if the two propellers are constructed alike and of the same material the stresses become alike and the deflections become equivalent too, which is a great additional advantage of model tests with full size tip speed.

In the variable density wind tunnel with compressed air, the velocity of sound will not be much changed, as it depends upon the temperature of the air only. Full tip velocity would therefore give the correct influence of the compressibility of the air, but the stresses and deflections would be too large. There could be made two tests with the same propeller model, one at the right Reynolds number and the right tip speed and the other with the right pressure distribution. If the scale of the model be 1/n, the pressure in the tunnel would have to be n atmospheres. Another test with the full tip velocity should be made at ordinary atmospheric pressure. Then both tests give the correct influence of the compressibility of the air, the one giving the influence of the viscosity, and the other the elasticity. This would give very instructive results. The combined effect of both properties of air can be sufficiently studied in flight.

With propeller model tests, the ratio of the tip velocity to the velocity of flight, has to be equal in the cases to be compared. That corresponds to an equal angle of attack of two airplanes. The rules (1) and (3) then hold with respect to an air force (thrust) and horsepower required.

Many airplane maneuvers are effected by acceleration of gravity, g; that is, if forces of gravity and forces of acceleration both occur. V^2/b is an expression having the dimension of an acceleration and hence, its ratio to g can be chosen as the absolute number, characteristic for the influence of gravity. Since the magnitude of the gravity g, does not vary, V^2/b has to be kept constant for maneuvers of different airplanes to be compared with respect to the influence of gravity. This leads to Froude's rule: the velocity has to be varied as the square root of the linear dimension.

$$V = c\sqrt{b} \qquad (6)$$

If Froude's rule is followed, the relations (1) to (5) can be transformed, as then one of the factors can be eliminated by the substitution of Froude's rule (6), resulting, for instance, in:

(7) Time $= c \sqrt{\text{length}} = c$ velocity,

(8) Force $= c\ b^3\ \rho$.

(9) $W/S = c\ b\ \rho$.

(10) $HP/S = c\ b^{3/2} \rho$.

III. I proceed now with the application of the model rules discussed, to the problem of two similar airplanes in flight. The influence of the viscosity and compressibility of the air can be disregarded in this case.

Steady flight.

With respect to steady flight, the influence of gravity can also be disregarded, requiring the use of the rules (1) to (5). The similarity is supposed to extend to an equal angle of attack, to equal control settings and to the propeller. The tip velocity of the propeller is then proportional to the velocity of flight.

With respect to a variation of the weight only (density and size content), equation (2) shows that the velocity decreases much more rapidly (equation (3)) as $W^{3/2}$. For instance, reducing the weight to 1/4 means reducing the velocity to 1/2 only, and the required horsepower is then but 1/8 of the original. These relations explain why low-powered and light, but not extremely small, airplanes are possible. They refer to any velocity, maximum or minimum, and rate of climb, and account for the good start and the comparatively high velocity of these light airplanes.

The start with the smallest horsepower as also the ceiling flight will take place at respectively equal angles of attack. If it can be assumed that the propeller torque is proportional to the density of air, and that the rate of revolution of the propeller at high altitudes does not become excessive when proportional to the speed of the airplane, the weights at different altitudes and equal angle of attack will become proportional to the density of the air. This follows directly from equations (1) to (5) together with the assumption regarding the variation of the propeller torque.

This relation can be used for determining the highest altitude at which an airplane can take off. The load of the airplane has to be increased until the pilot is just able to accomplish the take-off. Then the ratio of the lowest density of take-off to the density of the test is equal to the ratio of the standard weight (at which the airplane has to start at a higher altitude) to the maximum weight at starting. With a supercharged engine the torque decreases less and the test may give too unfavorable a result, except when the revolutions of the propeller are limited not to exceed the rate of revolution corresponding to this model rule.

With similar airplanes, of different weight and size, the velocity is proportional to the square root of the unit load, (equation (2)). The unit horsepower HP/S is proportional to $(W/S)^{3/2}$. Hence the velocity and the horsepower load HP/S do not necessarily change if the size of the airplane is changed.

It can be concluded from equation (4) that the time for any maneuver not influenced by gravity, is proportional directly to the linear dimension and inversely to the velocity. Certain types of oscillations have periods following this rule.

Unsteady maneuvers.

Most of the unsteady maneuvers of an airplane are influenced by gravity. Hence no direct conclusions can be drawn from one airplane in relation to another similar one, regarding such maneuvers, unless Froude's rule has been complied with. Hence, we have now to use equations (6) to (10).

If equations (9) and (1) are followed, conclusions can be drawn regarding the stability, controllability and maneuverability of the airplane. The time of any maneuver of the period of any oscillation will be proportional to the square root of the length or to the velocity (equation (7)). The path of the airplanes will be similar if the same maneuvers are made. The radius of shortest turn, for instance, will be proportional to a linear dimension of the airplane.

Froude's rule also includes the question as to how a seaplane can start from the water surface. It thus becomes possible to determine whether the starting of a giant seaplane can be accomplished by first building a small similar airplane complying with equations (9) and (10).

It is impossible to discuss in a short note all items which could be investigated through model tests. The general scheme to be followed is always the same. The factors governing the result give the conditions of the test to be complied with, and if so done the result can be converted proportional to any expression having the same physical dimension.

Conclusions, by comparison, can also be drawn if the airplanes are not exactly similar. This, however, requires much more judgment and experience, and the result will only be approximately correct. If the types are distinctly different, for instance, if the control surfaces are comparatively of very different size, there is ordinarily a fundamental reason for such difference, which becomes apparent when forming the absolute numbers by which the governing factors are connected. An additional factor, not yet mentioned in this note, may be, for instance, the absolute magnitude of the prevalent wind. In all such cases, the application of the theory of the physical dimensions and of the model rules derived therefrom, as pointed out in this note, will be enlightening and instructive.

NOTES ON GEOMETRICAL SIMILARITY IN AIRPLANES.

By Edward P. Warner.

The growing popularity of the light plane, and the repeated suggestions that it may prove a satisfactory vehicle for making preliminary tests from which the performance of much larger airplanes can be predicted, make it desirable that an investigation of the relation between large and small airplanes of geometrically similar form be undertaken. Already one prominent French airplane constructor has built a machine of but little above the light plane class as a scale model of a giant airplane which he has projected, and the construction of the large airplane will presumably be governed to some extent by the lessons learned during the trials of the small one. There can be no doubt, if this project of making man-carrying scale models proves a practicable one, that it will be very widely taken up.

Instead of seeking to establish perfectly general relations between the performances of a large airplane and a small one, it seems desirable rather to determine the ratios which should exist between certain geometrical characteristics in order that the performances may stand in some particular desired relation. The elements of performance, including all the flying qualities of the airplane within that term, are measurable in terms of length, time and angle, as fundamental quantities, those quantities appearing either singly or in combination. Obviously, those elements of performance measured in terms of angle, such as the inclination of the climbing path, should be independent of the size of the airplane, while those measured in length, such as the minimum radius of turn, should be directly proportional to the linear dimension. The question of the variation of those elements into which time enters may be put aside for the moment, except for mention of the obvious fact that the linear velocities must be proportional to the product of angular velocities and linear dimension.

The various elements of performance and the characteristics of the airplane will now be taken up and investigated one by one. It would, of course, be possible, to rely on the general theory of dimensions in deriving the desired relations, but conclusions can be extended over a somewhat broader range if each feature of performance is analyzed separately by methods simpler, and in some cases less rigorous, than the general theory.

Minimum Radius of Turn.

Centripetal force in turning is proportional to the product of weight, angular velocity, and linear velocity. If airplanes of different size are to turn at the same angle of bank and with the same control setting the ratio between centripetal force and weight must obviously be constant, and the product of angular and linear velocities must therefore be independent of linear dimension. The square of the speed must then be proportional to the radius of the turn for a given condition of flight. Since the radius is a distance, it should vary as the first power of a dimension of the airplane, if strict similarity of performance is maintained, and the speed must therefore be proportional to the square root of such a dimension of the airplane. It follows also that the angular velocity is inversely proportional to the square root of a linear dimension. Since the loading of the wings varies directly as the square of the speed for a given angle of attack, $\frac{W}{S}$ must be proportional to the first power of a dimension of the airplane, and the total weight must vary as the cube of such a dimension. For similarity of performance there is then the same rule of variation of weight as for strictly geometrically similar structures.

If this relation of weight and size be followed, not only the minimum radius of turn, but also the radius for any specified set of conditions will be proportional to the first power of a length in the airplane. For any fixed angle of attack and angle of bank the ratio of turning radius to wing span will be independent of size.

Controllability in Turning.

The maximum angular velocity of an airplane is sometimes fixed by the power of the control to overcome the damping of the rotation, rather than by a simple balancing of centripetal force and horizontal component of lift. Damping moments are proportional to the product of an angular velocity, a linear velocity, and the fourth power of a linear dimension, while control moments vary as the cube of a length and the square of a linear speed. In order that controlling power may enter in as a limitation in the same way for a whole series of geometrically similar airplanes, it is therefore sufficient that $l^3 V^2 = l^4 V \omega$ or, that $V = \omega l$. Obviously, any relation between speed and size which will make radius of turn proportional to length of airplane will also satisfy this equation. The constant ratio between radius and length will therefore hold good, no matter what the factor principally limiting radius may be.

Angular Acceleration.

The angular acceleration of an airplane for a given control setting is of course proportional to the ratio of controlling moment to moment of inertia. The first of these quantities varies as the cube of a linear dimension and the square of a speed, the second as the product of the weight and the square of a length, and the ratio is therefore proportional to $\frac{V^2 l}{W}$. If the relation, already derived, between l, V, and W be complied with, this varies inversely as a length. The time required to reach a specified velocity at constant angular acceleration would therefore be proportional to the linear dimension of the airplane, but, since the maximum angular velocity itself varies inversely as the square root of a length, the time required to reach the maximum is proportional only to $\sqrt{l}$. Distance being the product of time and speed, the distance covered in reaching maximum angular velocity or any particular fraction of the maximum must be directly proportional to l, and the angle through which the airplane turns from the beginning of a maneuver until a particular proportion of the maximum attainable angular velocity has been arrived at is quite independent of dimension. This, again, is as it should be.

Dynamic Stability.

While on the subject of control and maneuvering power, attention may be given also to the equations determining the amplitude and period of the oscillations of an airplane. The work need not be followed in detail, but, if the variation of each of the resistance and rotary derivatives be examined separately, it is found that the coefficients in the familiar stability equation:

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

vary with l in a descending scale of powers of $l^{1/2}$, the first coefficient varying as l^2. This is obviously equivalent in effect on any solution of the equation to a variation of λ, the logarithmic determinant itself, in the ratio of the inverse square root of l. Since the amplitude of an oscillation at any time subsequent to its beginning is equal to $Ce^{\lambda t}$,* the time required to damp or increase the amplitude of oscillation by a definite ratio must obviously be proportional to $\sqrt{l}$; if λ varies as the inverse square root. V and t then change with l in the same way, while the distance required to damp an oscillation by a specified amount or to complete one period of an oscillation is proportional to the product of V and t, or to a linear dimension of the airplane. Thus, once again, two lengths vary in the same ratio.

Minimum Speed.

The minimum speed, being proportional to the square root of the wing loading, evidently varies as $\sqrt{l}$. The kinetic energy possessed by the airplane on coming in contact with the ground then varies as l^4, and, assuming the coefficient of friction the same in all cases, the landing run during which friction and air resistance dissipate this kinetic energy varies directly as a linear dimension of the airplane.

Maximum Speed.

Obviously, in order that performances may be comparable, geometrically similar airplanes should fly at maximum speed at the same angle of attack. The maximum speeds must therefore vary in the same ratio as the minima. At a given angle of attack the power required for flight is proportional to the product of the area and the cube of the speed. If the speed varies as $\sqrt{l}$, and the propeller efficiency is constant, the power must then be proportional to $l^{7/2}$. To satisfy maximum speed requirements in a series of geometrically similar airplanes the engine power must therefore vary somewhat more rapidly than the weight.

Propeller Efficiency.

In order that propeller efficiency may be constant, the slip function $\frac{V}{N D}$ must be held at a constant value. If V varies as $l^{1/2}$, N must therefore be inversely proportional to $\sqrt{l}$. This condition satisfies the equation of propeller power absorption also. Since the power consumed by a propeller at a constant value of the slip function is proportional to $N^3 D^5$, it will vary as $l^{7/2}$, if N changes with size in the manner just stated.

Speed and Angle of Climb.

The ratio between the powers required for flight at two particular angles of attack is obviously independent of airplane size. If the maximum speed corresponds to the same angle of attack in every case and the propeller characteristics are in accordance with the relation just derived, the percentage of reserve power at the angle of best climb will then be the same for a whole series of geometrically similar airplanes. Dividing reserve power by weight it appears that climbing speed varies as $l^{1/2}$, or, in the same ratio as speed of flight. The climbing angle is therefore the same in all cases.

Linear Acceleration.

The linear acceleration of an airplane in taking off is proportional to the ratio of thrust to weight, and that is obviously constant, if the prescribed relation between power and other characteristics of the airplane be preserved. The distance traveled in acquiring a given velocity is then proportional to the square of this velocity, and that, in turn, is proportional to l.

It has now been seen that all of the flying qualities of a small airplane can be made directly comparable with those of a large one, if a very simple relation between size, weight, power, and R.P.M. is maintained. That could have been predicted from the general theory of dimensions, following the line of Dr. Munk's work, and, indeed, the relations derived are identical with those given by Froude's law of comparison, and used for ships.* There are some points, however, at which similarity of performance breaks down. As pointed out by Dr. Munk, the condition of aerodynamic similitude, which would make the speed inversely proportional to a linear dimension, cannot be maintained, and the relation existing between flying characteristics in large and small sizes will also be modified by any structure in the atmosphere, a structure which will necessarily have linear dimensions of its own. Either periodic gusts or regions of turbulence will have effects depending largely on the size of the airplane which meets them. It is therefore somewhat unsafe to attempt to predict the behavior of a giant airplane in rough air from tests on a miniature prototype, but there need be no hesitancy about the application of data thus obtained on performance and on maneuverability under good conditions. The variation of the Reynolds number is unlikely to have any serious effect after values even as large as those for the smallest of light planes have been reached.

* Speed and Power of Ships, by D. W. Taylor, p.26.

Structural Relations.

The fact that it has been found necessary to vary the weight as the cube of a linear dimension suggests the possibility of building the structures in strict geometrical similarity, in order that the percentage of weight allotted to each part may remain the same in all cases. That would, indeed, be highly desirable for complete similarity of performance, as the radii of gyration are hardly likely to vary in the same manner as the overall dimensions, unless all the internal structure is kept of similar form as size is changed.

It is, of course, impossible to hold rigidly to similarity of structure. The thickness of fabric, for example, can hardly be decreased in proportion to the wing span, and the type of joint used in built-up members of large airplanes can hardly be duplicated in small ones. To a certain point, however, similarity can be maintained if it proves to be structurally safe to hold to it.

Considering first those members which are loaded directly in tension or compression, it is obvious that their strength is proportional to l^2. This is true even of long struts, since the ratio of l to k will be independent of the size of the airplane. The load carried by such members is proportional to the airplane weight, and the factor of safety in them therefore varies inversely as a linear dimension. Over the range of sizes now used, this is just about the desirable rate of variation, as it will be found that the load factors now specified for a high angle of attack are given approximately for all classes of military airplanes by the formula: $F = \frac{330}{b}$, where b is the wing span.

A similar relation holds true for beams. The bending moment varies as the weight of the airplane times the span, or as the fourth power of a linear dimension, while the section modulus is proportional to l^3. The factor of safety at a given load factor again changes inversely with l. When the beam is subject to buckling, however, the relation is no longer simple. The column effect is approximately allowed for by Perry's formula: $M' = M \times \frac{P_e}{P_{e-P}}$, where M is the bending moment due to lateral load, M' the bending moment corrected for buckling, P the compressive load, and P_e the load which would produce failure by lateral collapse if there were no lateral load at all. P is proportional to l^2, P_e to l^3, and the corrected bending moment under unit load therefore changes with linear dimension in an irregular fashion. If, however, the load factor assumed to act itself varies as the inverse first power of l, P_e and P will change at the same rate and the column effect will remain always of the same relative importance.

It is also of interest at times to know the deflection of the parts of an airplane. The flexural deflection of the wing spars, being proportional to $\frac{Wl^3}{EI}$, will vary as l^2, if the spars are made in the same way and of the same material. Deflections of the wing truss due to the direct elongation and compression of the members also follow the same law, since the unit stress under a given load factor has been shown to be proportional to l and the total change of length of each member must therefore be in the ratio of l^2. If, however, the highest load factors actually imposed are in the ratio of $\frac{1}{l}$ the structures will deflect in a geometrically similar manner.

Deflection is perhaps most serious in its effect on the performance of the propeller, the angle of twist of the propeller blade being proportional to l in geometrically similar airplanes. A type of propeller suitable for a small airplane might therefore be quite unsatisfactory on a large one of the same design, even though its calculated strength were sufficient, and tests on geometrically similar airplanes should be carried out with propellers so designed as to have a minimum of torsion.

Illustrative Example.

To show how all of this work can be applied in practice, an airplane similar in general characteristics to the Barling Bomber, having a total weight of 42,000 pounds, a span of 130 feet, an area of 4200 square feet, and powered with six 400 HP engines may be used as an example. Models of one-third, one-fourth, and one-fifth full size have been calculated, and their characteristics are tabulated below:

Span ft.	Area sq.ft.	Weight lb.	Total horsepower	R.P.M.
43	467	1560	52	2950
32	262	660	19	3400
26	168	340	8.5	3800

Obviously, the third case is impossible to realize, as the pilot's weight would be more than half of the total carried in flight, and the six engines of $1\frac{1}{2}$ horsepower each would make up most of the remainder. The second case might be barely possible with 3 horsepower engines specially built for the purpose. The weight available for structure would be about 300 pounds, the area being 260 square feet, and the wing loading 2.5 pounds per square foot. The first case would be easy to realize.

With everything considered, and the advantage and drawbacks of the light plane as a flying model balanced against each other, it still seems quite possible that the construction of such flying models would be well worth while in some cases, particularly if the development of large airplanes of eccentric form and arrangement is to continue, and the practice initiated by the French constructor, already referred to, may on occasion prove a profitable one elsewhere.